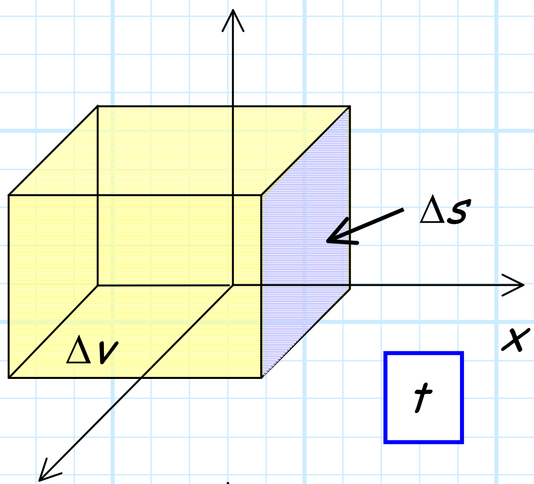
Charge Velocity and Current Density

Consider a **small** volume (Δv) filled with charge Q.

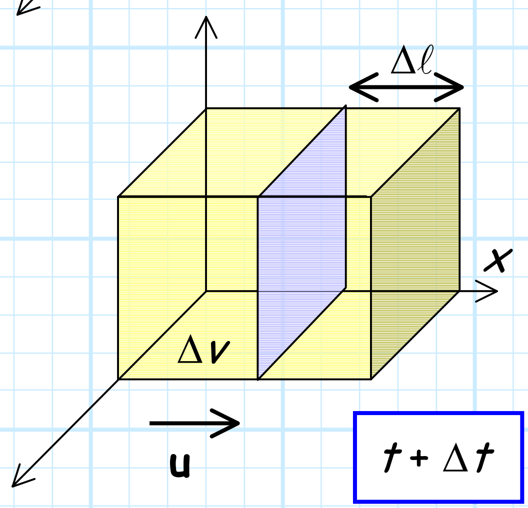
If the charge is **uniformly** distributed, then the **charge density** is:

$$\rho_v(\bar{\mathbf{r}}) = \frac{Q}{\Delta v}$$



Say these charges are **moving** at velocity $\mathbf{u} = u_x\, \hat{a}_x$. Then, in a small **time** Δt, the charged particles will have moved in the x-direction a **distance** $\Delta \ell$:

$$\Delta \ell = u_x\, \Delta t$$



Q: *How much **charge** ΔQ moves across **surface** Δs in **time** Δt ?*

A: The amount is **equal** to the charge occupying **volume** $\Delta s\, \Delta \ell$:

$$\Delta Q = \rho_v(\bar{r})(\Delta s\,\Delta\ell)$$

But remember, $\Delta\ell = u_x\,\Delta t$. Therefore:

$$\Delta Q = \rho_v(\bar{r})\,u_x\,\Delta s\,\Delta t$$

And dividing by Δt:

$$\frac{\Delta Q}{\Delta t} = \rho_v(\bar{r})\,u_x\,\Delta s$$

Hey! Charge divided by time is equal to **current** !

$$\Delta I = \frac{\Delta Q}{\Delta t} = \rho_v(\bar{r})\,u_x\,\Delta s$$

The current ΔI is the current flowing **through** the small surface Δs. We can therefore determine the **current density** on this surface:

$$J_x = \frac{\Delta I}{\Delta s} = \rho_v(\bar{r})\;u_x$$

In other words, current density is equal to the **product** of the charge density and the charge velocity. In general, we can say:

$$\mathbf{J}(\bar{r}) = \rho_v(\bar{r})\,\mathbf{u}(\bar{r})$$

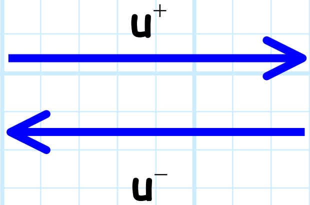
where $\mathbf{u}(\bar{r})$ is a vector field that describes the **velocity** of the moving charge at every point $\bar{r}$.

IMPORTANT NOTE! The velocity of charge is **NOT** the speed of light! In fact, charge velocity is generally **nowhere** near $c = 3 \times 10^8$ m/sec (its more like 3×10^{-2} m/sec!).

Charge velocity is generally dependent on the **type** of particles that carry the charge (e.g., free electrons, positive ions).

For example, we can denote $\mathbf{u}^+$ the velocity of **positively** charged particles, while $\mathbf{u}^-$ denotes the velocity of **negatively** charged particles.

We find that typically, $\mathbf{u}^+$ and $\mathbf{u}^-$ point in **opposite** directions!



and the velocities will have **unequal** magnitudes:

$$|\mathbf{u}^+| \neq |\mathbf{u}^-|$$

The total current density can therefore be expressed as:

$$\begin{aligned}\mathbf{J}(\bar{r}) &= \mathbf{J}^+(\bar{r}) + \mathbf{J}^-(\bar{r}) \\ &= \rho_v^+(\bar{r})\,\mathbf{u}^+(\bar{r}) + \rho_v^-(\bar{r})\,\mathbf{u}^-(\bar{r})\end{aligned}$$

Q: *So, $\mathbf{J}^{+}(\bar{r})$ and $\mathbf{J}^{-}(\bar{r})$ must point in opposite directions, since $\mathbf{u}^{+}(\bar{r})$ and $\mathbf{u}^{-}(\bar{r})$ point in opposite directions ?*

A: NO! It is true that the charges flow in opposite **directions**, but the charges also have opposite **signs** ! Recall $\rho_v^{+}(\bar{r}) > 0$ and $\rho_v^{-}(\bar{r}) < 0$, therefore, vectors $\mathbf{J}^{+}(\bar{r}) = \rho_v^{+}(\bar{r})\mathbf{u}^{+}(\bar{r})$ and $\mathbf{J}^{-}(\bar{r}) = \rho_v^{-}(\bar{r})\mathbf{u}^{-}(\bar{r})$ each typically point in the **same** direction!

Remember that, for example, if positive charge is moving **left** and negative charge is moving **right**, then **both** result in **current** flowing toward the **left**.